

## AN EXTENDED HARDY–HILBERT’S INEQUALITY WITH PARAMETERS AND APPLICATIONS

ZHAOHUI GU AND BICHENG YANG\*

**Abstract.** By using the weight coefficients, the Euler-Maclaurin summation formula and Abel’s summation by parts formula, an extended Hardy-Hilbert’s inequality with the power function as the interval variables and a new Hilbert-type inequality with the partial sums are given. As applications, the equivalent conditions of the best possible constant factor in a particular inequality related to a few parameters and some particular cases are considered.

**Mathematics subject classification (2020):** 26D15.

**Keywords and phrases:** Weight coefficient, Euler-Maclaurin summation formula, Hardy-Hilbert’s inequality, internal variable, parameter, partial sum.

### REFERENCES

- [1] G. H. HARDY, J. E. LITTLEWOOD AND G. POLYA, *Inequalities*, Cambridge University Press, Cambridge, 1934.
- [2] M. KRNIĆ AND J. PEČARIĆ, *Extension of Hilbert’s inequality*, J. Math. Anal., Appl. 324 (1), 150–160 (2006).
- [3] B. YANG, *On a generalization of Hilbert double series theorem*, J. Nanjing Univ. Math. Biquarterly, 18 (1), 145–152, 2001.
- [4] V. ADIYASUREN, T. BATBOLD, L. E. AZAR, *A new discrete Hilbert-type inequality involving partial sums*, Journal of Inequalities and Applications, 2019:127, 2019.
- [5] B. C. YANG, *The norm of operator and Hilbert-type inequalities*, Science Press, Beijing, China, 2009.
- [6] M. KRNIĆ AND J. PEČARIĆ, *General Hilbert’s and Hardy’s inequalities*, Mathematical inequalities & applications, 8 (1), 29–51 (2005).
- [7] I. PERIĆ AND P. VUKOVIĆ, *Multiple Hilbert’s type inequalities with a homogeneous kernel*, Banach Journal of Mathematical Analysis, 5 (2), 33–43 (2011).
- [8] Q. L. HUANG, *A new extension of Hardy-Hilbert-type inequality*, Journal of Inequalities and Applications (2015), 2015:397.
- [9] B. HE, *A multiple Hilbert-type discrete inequality with a new kernel and best possible constant factor*, Journal of Mathematical Analysis and Applications, 431, 990–902 (2015).
- [10] J. S. XU, *Hardy-Hilbert’s inequalities with two parameters*, Advances in Mathematics, 36 (2), 63–76 (2007).
- [11] Z. T. XIE, Z. ZENG AND Y. F. SUN, *A new Hilbert-type inequality with the homogeneous kernel of degree  $-2$* , Advances and Applications in Mathematical Sciences, 12 (7), 391–401 (2013).
- [12] Z. ZHEN, K. RAJA RAMA GANDHI AND Z. T. XIE, *A new Hilbert-type inequality with the homogeneous kernel of degree  $-2$  and with the integral*, Bulletin of Mathematical Sciences and Applications, 3 (1), 11–20 (2014).
- [13] D. M. XIN, *A Hilbert-type integral inequality with the homogeneous kernel of zero degree*, Mathematical Theory and Applications, 30 (2), 70–74 (2010).
- [14] L. E. AZAR, *The connection between Hilbert and Hardy inequalities*, Journal of Inequalities and Applications, 2013:452, 2013.
- [15] V. ADIYASUREN, T. BATBOLD AND M. KRNIĆ, *Hilbert-type inequalities involving differential operators, the best constants and applications*, Math. Inequal. Appl., 18, 111–124 (2015).

- [16] M. TH. RASSIAS, AND B. C. YANG, *On half-discrete Hilbert's inequality*, Applied Mathematics and Computation, 220, 75–93 (2013).
- [17] B. C. YANG AND M. KRNIĆ, *A half-discrete Hilbert-type inequality with a general homogeneous kernel of degree 0*, Journal of Mathematical Inequalities, 6 (3), 401–417 (2012).
- [18] M. TH. RASSIAS AND B. C. YANG, *A multidimensional half – discrete Hilbert-type inequality and the Riemann zeta function*, Applied Mathematics and Computation, 225, 263–277 (2013).
- [19] M. TH. RASSIAS AND B. C. YANG, *On a multidimensional half-discrete Hilbert-type inequality related to the hyperbolic cotangent function*, Applied Mathematics and Computation, 242, 800–813 (2013).
- [20] B. C. YANG AND L. DEBNATH, *Half-discrete Hilbert-type inequalities*, World Scientific Publishing, Singapore, 2014.
- [21] Y. HONG AND Y. WEN, *A necessary and sufficient condition of that Hilbert type series inequality with homogeneous kernel has the best constant factor*, Annals Mathematica, 37A (3), 329–336 (2016).
- [22] Y. HONG, *On the structure character of Hilbert's type integral inequality with homogeneous kernel and application*, Journal of Jilin University (Science Edition), 55 (2), 189–194 (2017).
- [23] Y. HONG, Q. L. HUANG, B. C. YANG AND J. L. LIAO, *The necessary and sufficient conditions for the existence of a kind of Hilbert-type multiple integral inequality with the non-homogeneous kernel and its applications*, Journal of Inequalities and Applications (2017), 2017:316.
- [24] D. M. XIN, B. C. YANG AND A. Z. WANG, *Equivalent property of a Hilbert-type integral inequality related to the beta function in the whole plane*, Journal of Function Spaces, Volume 2018, Article ID 2691816, 8 pages.
- [25] Y. HONG, B. HE AND B. C. YANG, *Necessary and sufficient conditions for the validity of Hilbert type integral inequalities with a class of quasi-homogeneous kernels and its application in operator theory*, Journal of Mathematics Inequalities, 12 (3), 777–788 (2018).
- [26] Z. X. HUANG AND B. C. YANG, *Equivalent property of a half-discrete Hilbert's inequality with parameters*, Journal of Inequalities and Applications (2018) 2018:333.
- [27] B. C. YANG, S. H. WU AND A. Z. WANG, *On a reverse half-discrete Hardy-Hilbert's inequality with parameters*, Mathematics, 2019, 7, 1054.
- [28] A. Z. WANG, B. C. YANG AND Q. CHEN, *Equivalent properties of a reverse's half-discret Hilbert's inequality*, Journal of Inequalities and Applications (2019), 2019:279.
- [29] X. S. HUANG, R. C. LUO AND B. C. YANG, *On a new extended Half-discrete Hilbert's inequality involving partial sums*, Journal of Inequalities and Applications (2020) 2020:16.
- [30] B. C. YANG, S. H. WU AND Q. CHEN, *On an extended Hardy-Littlewood-Polyá's inequality*, AIMS Mathematics, 5 (2), 1550–1561 (2020).
- [31] J. Q. LIAO, S. H. WU AND B. C. YANG, *On a new half-discrete Hilbert-type inequality involving the variable upper limit integral and the partial sum*, Mathematics, 2020, 8, 229; doi:10.3390/math8020229.
- [32] B. C. YANG, M. F. HUANG AND Y. R. ZHONG, *Equivalent statements of a more accurate extended Mulholland's inequality with a best possible constant factor*, Mathematical Inequalities and Applications, 23 (1), 231–244 (2020).
- [33] B. C. YANG, S. H. WU AND A. Z. WANG, *A new Hilbert-type inequality with positive homogeneous kernel and its equivalent form*, Symmetric, 2020, 12, 342, doi:10.3390/sym12030342.
- [34] Z. X. HUANG, Y. P. SHI AND B. C. YANG, *On a reverse extended Hardy-Hilbert's Inequality*, Journal of Inequalities and Applications (2020), 2020:68.
- [35] M. TH. RASSIAS, B.C. YANG AND A. RAIGORODSKII, *On Hardy-type integral inequality in the whole plane related to the extended Hurwitz-zeta function*, Journal of Inequalities and Applications (2020), 2020:94.
- [36] J. Q. LIAO, Y. HONG AND B. C. YANG, *Equivalent conditions of a Hilbert-type multiple integral inequality holding*, Journal of Function Spaces, Volume 2020, Article ID 3050952, 6 pages.
- [37] A. Z. WANG AND B. C. YANG, *Equivalent property of a more accurate half-discrete Hilbert's inequality*, Journal of Applied Analysis and Computation, 10 (3), 920–934 (2020).
- [38] Y. HONG, J. Q. LIAO, B. C. YANG AND Q. CHEN, *A class of Hilbert-type multiple integral inequalities with the kernel of generalized homogeneous function and its applications*, Journal of Inequalities and Applications (2020), 2020:140.
- [39] B. C. YANG, S. H. WU AND Q. CHEN, *A new extension of Hardy-Hilbert's inequality containing kernel of double power functions*, Mathematics, 2020, 8, 339; doi:10.3390/math8060894.

- [40] J. C. KUANG, *Applied inequalities*, Shangdong Science and Technology Press, Jinan, China, 2004.