

EQUIVALENT CONDITIONS OF OPTIMAL HALF DISCRETE HILBERT TYPE MULTIPLE INTEGRAL INEQUALITIES WITH QUASI HOMOGENEOUS KERNEL AND APPLICATIONS

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Abstract. Let $G(u, v)$ be a λ -order homogeneous function. In this paper, by discussing optimal matching parameters of the half discrete Hilbert type multiple integral inequality with quasi-homogeneous kernel $K(n, \|x\|_{\rho, m}) = G(n^{\lambda_1}, \|x\|_{\rho_2, m}^{\lambda_2})$, several equivalent conditions of the optimal matching parameters are obtained, and a basic theoretical problem of the half discrete Hilbert type inequality is solved. Finally, their applications to operator boundedness and operator norm are discussed.

Mathematics subject classification (2020): 26D15, 47A07.

Keywords and phrases: Quasi-homogeneous kernel, half discrete Hilbert type multiple integral inequality, the best constant factor, optimal matching parameter, equivalent condition, operator norm.

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