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SOME DEGENERATE MEAN CONVERGENCE THEOREMS FOR BANACH SPACE VALUED RANDOM ELEMENTS

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Abstract. For an array $\{V_{n,j}, 1 \leqslant j \leqslant k_n, n \geqslant 1\}$ of random elements taking values in a real separable Rademacher type p $(1 < p \leqslant 2)$ Banach space and a sequence of positive constants $\{d_n, n \geqslant 1\}$, a theorem is established providing conditions under which the degenerate mean convergence result $\mathbb{E}\|(S_n - \mathbb{E}S_n)/d_n\|^p \to 0$ holds where $S_n = \sum_{j=1}^{n} V_{n,j}$, $n \geqslant 1$. An example is provided showing that the above degenerate mean convergence can fail if the Banach space is not of Rademacher type p where $1 < p \leqslant 2$. Moreover for a general sequence of random elements $\{W_n, n \geqslant 1\}$ which is not structurally of any specific form taking values in a real separable Banach space which is not assumed to be of Rademacher type p for any $p \in (1,2]$, conditions are provided under which the degenerate mean convergence result $\mathbb{E}(g(\|W_n\|)) \to 0$ holds where g is a continuous strictly increasing function with $g(0) = 0$ and $\lim_{x\to\infty} g(x) = \infty$.

Mathematics subject classification (2020): 60F25, 60B11, 60B12.
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