SUPPORTING VECTORS FOR THE ℓ_p–NORM

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Abstract. Given a continuous linear operator $T : X \to Y$ between normed spaces X, Y, the set of supporting vectors of T is defined as $\mathrm{suppv}(T) := \{x \in X : \|T(x)\| = \|T\| \text{ and } \|x\| = 1\}$. The supporting vectors of nontrivial projections and operators on ℓ_p, for $p = 1, 2, \infty$, have already been calculated in previous works. In this manuscript, we go on one step further and compute the supporting vectors of operators $T : \ell_p \to \ell_q$, where $1 < p, q < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$.

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