

## MORE RESULTS ON WEIGHTED MEANS

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*Abstract.* We give a refined Young inequality which generalizes the inequality by Zou–Jiang. We also show the upper bound for the logarithmic mean by the use of the weighted geometric mean and the weighted arithmetic mean. Furthermore, we show some inequalities among the weighted means. Based on the obtained essential scalar inequalities, we give some operator inequalities. In particular, we give a generalization of the result by Zou–Jiang, that is, we show the operator inequalities with the operator relative entropy with the weighted parameter. Finally, we give the further generalized inequalities by the Tsallis operator relative entropy.

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