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LOWER DIMENSIONAL ELLIPSOIDS OF MAXIMAL VOLUME IN CONVEX BODIES

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Abstract. In this paper, we show that the volume of a k-dimensional ellipsoid in the convex body formed by centered isotropic measures on the unit sphere is no large than that of a k-dimensional Ball of radius $\sqrt{n(n+1)/k(k+1)}$. It generalizes the John theorem to the lower dimensional cases.

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REFERENCES

[1] K. BALL, *An elementary introduction to modern convex geometry*, Flavors of geometry, Math. Sci. Res. Inst. Publ., 31, Cambridge University Press, Cambridge, 1997, 1–58.
[2] K. BALL, *Volumes of sections of cubes and related problems*, Israel seminar on Geometric Aspects of Functional Analysis, number 1376 in Lectures Notes in Mathematics. Springer-Verlag, 1989.
[3] K. BALL, *Volume ratios and a reverse isoperimetric inequality*, J. London Math. Soc. **44** (1991), 351–359.
[4] K. BALL, *Ellipsoids of maximal volume in convex bodies*, Geom. Dedicata **41** (1992), 241–250.
[5] J. BASTERO, M. ROMANCE, *John's decomposition of the identity in the non-convex case*, Positivity **6** (2002), 1–161.
[6] J. BASTERO, J. BERNUÉS AND M. ROMANCE, *From John to Gauss-John positions via dual mixed volumes*, J. Math. Anal. Appl. **328** (2007), 550–566.
[7] A. GIANNOPOULOS, V. D. MILMAN, *Extremal problems and isotropic positions of convex bodies*, Israel J. Math. **117** (2000), 29–60.
[8] A. GIANNOPOULOS, I. PERISSINAKI AND A. TSOLOMITIS, *John's theorem for an arbitrary pair of convex bodies*, Geom. Dedicata **84** (2001), 63–79.
[9] Y. GORDON, A. E. LITVAK, M. MEYER, AND A. PAJOR, *John's decomposition in the general case and applications*, J. Differential Geom. **68** (2004), 99–119.
[10] P. M. GRUBER, *Application of an idea of Voronoi to John type problems*, Adv. Math. **218** (2008), 309–351.
[11] P. M. GRUBER, F. SCHUSTER, *An arithmetic proof of John's ellipsoid theorem*, Arch. Math. **85** (2005), 82–88.
[12] D. HENSLEY, *Slicing the cube in $\mathbb{R}^n$ and probability*, Proc. Amer. Math. Soc. **73** (1979), 95–100.
[13] F. JOHN, *Extremum problems with inequalities as subsidiary conditions*, Studies and Essays Presented to R. Courant on his 60th Birthday, January 8, 1948, 187–204, Interscience Publishers, Inc., New York, NY, 1948.
[14] D. LEWIS, *Ellipsoids defined by Banach ideal norms*, Mathematika **26** (1979), 18–29.
[15] A.-J. LI, G. LENG, *Brascamp-Lieb inequality for positive double John basis and its reverse*, Science in China, Series A **54** (2011), 399–410.
[16] G. PISIER, *The volume of convex bodies and Banach space geometry*, Cambridge Tracts in Mathematics, 94, Cambridge, Cambridge University Press, 1989.
[17] N. TOMCZAK-JAEGERMANN, *Banach-Mazur distances and finite-dimensional operator ideals*, Pitman Monographs and Surveys in Pure and Applied Mathematics, 38, Pitman, London, 1989.