OPTIMAL CRITICAL EXPONENT L^p INEQUALITIES OF HARDY TYPE ON THE SPHERE VIA XIAO'S METHOD

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Abstract. First, we correct the proof presented in [Sharp L^p Hardy type and uncertainty principle inequalities on the sphere, Journal of Mathematical Inequalities, 13, 4 (2019), 1011–1022] and obtain a sharp version of an L^p Hardy inequality on the sphere $\mathbb{S}^n$ for all $2 \leqslant p < n$. Secondly, we prove sharp critical exponent L^n inequalities on the sphere $\mathbb{S}^n$ in $\mathbb{R}^n$, $n \geqslant 2$. The singularity in this problem is the geodesic distance from an arbitrary point on the sphere. Moreover, we prove that neither the L^p Hardy inequality has a nontrivial maximizer in $W^{1,p}(\mathbb{S}^n)$ for any $2 \leqslant p < n$, nor does the limiting case L^n Hardy inequality have a nontrivial maximizer in $W^{1,n}(\mathbb{S}^n)$.

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