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WEAKLY SINGULAR HENRY–GRONWALL–BIHARI TYPE INEQUALITIES AND THEIR APPLICATIONS

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Abstract. In this work, we establish some new weakly singular Henry-Gronwall-Bihari type inequalities that are generalized from some recent works. Unlike most previous papers, in this work, we consider integral inequalities that include two integrals with doubly singular kernels, and obtain the bounds by an exponential function. We apply the obtained results to investigate the existence and uniqueness of solution of a fractional differential equation and a class of integral equations with weakly singular sources.

Mathematics subject classification (2020): 26D10, 26A33, 34A08, 34A12.
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