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ON AN INEQUALITY FOR 3–CONVEX FUNCTIONS AND RELATED RESULTS

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Abstract. Majorization type theorems (such as the Karamata inequality, the Fuchs inequality) for higher convex functions are rare and the criteria given in these theorems are difficult to check (see [10, Chapter 9.]). On the other side, the Brady theorem (see [3]) gives rather simple and the straightforward criterion for such type of results. We apply Brady's theorems on inequalities originated from 3-exponential convexity of certain function and, as a by-product, we obtain improvements of AG-inequality and an interesting mean. As an equivalent version of Brady's theorem, the mean value theorems, which are usually used in the definition of Stolarsky means, are also proved.

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