

SOME NOTES ON THE INCLUSION BETWEEN MORREY SPACES

PHILOTHEUS E. A. TUERAH AND NICKY K. TUMALUN*

Abstract. In this paper, we show that the Morrey spaces $\mathcal{M}_{q_1}^p(\mathbb{R}^n)$ cannot be contained in the weak Morrey spaces $w\mathcal{M}_{q_2}^p(\mathbb{R}^n)$ for $q_1 \neq q_2$. We also show that the vanishing Morrey spaces $\mathcal{V}\mathcal{M}_q^p(\mathbb{R}^n)$ are not empty and properly contained in the Morrey spaces $\mathcal{M}_q^p(\mathbb{R}^n)$.

Mathematics subject classification (2020): 46E30, 42B35.

Keywords and phrases: Morrey spaces, vanishing Morrey spaces.

REFERENCES

- [1] C. B. MORREY, *On the solutions of quasi-linear elliptic partial differential equations*, Trans. Amer. Math. Soc., **43**, (1938), 126–166.
- [2] C. VITANZA, *Functions with vanishing Morrey norm and elliptic partial differential equations*, in Proceedings of Methods of Real Analysis and Partial Differential Equations, Capri, (1990).
- [3] D. I. HAKIM AND Y. SAWANO, *Complex interpolation of various subspaces of Morrey spaces*, Sci. China Math., **63**, 5 (2020), 937–964.
- [4] H. GUNAWAN, D. I. HAKIM, AND M. IDRIS, *Proper inclusions of Morrey spaces*, Glasnik Matematički., **53**, 1 (2018), 143–151.
- [5] H. GUNAWAN, D. I. HAKIM, K. M. LIMANTA, AND A. A. MASTA, *Inclusion properties of generalized Morrey spaces*, Math. Nachr., **290**, 1 (2017), 332–340.
- [6] H. GUNAWAN, D. I. HAKIM, E. NAKAI, AND Y. SAWANO, *On inclusion relation between weak Morrey spaces and Morrey spaces*, Nonlinear Analysis., **168**, (2018), 27–31.
- [7] N. K. TUMALUN AND H. GUNAWAN, *Morrey spaces are embedded between weak Morrey spaces and Stummel classes*, J. Indones. Math. Soc., **253**, (2019), 203–209.
- [8] N. K. TUMALUN, D. I. HAKIM, AND H. GUNAWAN, *Inclusion between generalized Stummel classes and other function spaces*, Math. Inequal. Appl., **232**, (2020), 547–562.
- [9] N. SAMKO, *Maximal, potential and singular operators in vanishing generalized Morrey spaces*, J. Global. Optim., **57**, (2013), 1385–1399.