

NEW ORDERING RELATIONS FOR THE HEINZ MEANS VIA HYPERBOLIC FUNCTIONS

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Abstract. In this paper, we create new types of upper and lower bounds of Heinz means using simplified relations based on hyperbolic functions. In particular, for any strictly positive operators $A, B \in B(\mathcal{H})$, we obtain the inequality

$$A\#_{2\mu-\tau}B + A\#_{1-\tau}B < 2H_{\tau}(A, B) < A\#_{\tau}B + A\#_{1-(2\mu-\tau)}B,$$

where $0 < A < B$ and $0 < \mu < \tau < 1$.

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