FIRST EIGENVALUE OF THE LAPLACIAN OF A GEODESIC BALL AND AREA–BASED SYMMETRIZATION OF ITS METRIC TENSOR

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Abstract. Given a Riemmanian manifold, we provide a new method to compute a sharp upper bound for the first eigenvalue of the Laplacian for the Dirichlet problem on a geodesic ball of radius less than the injectivity radius of the manifold. This upper bound is obtained by transforming the metric tensor into a rotationally symmetric metric tensor that preserves the area of the geodesic spheres. The provided upper bound can be computed using only the area function of the geodesic spheres contained in the geodesic ball and it is sharp in the sense that the first eigenvalue of geodesic ball coincides with our upper bound if and only if the mean curvature pointed inward of each geodesic sphere is a radial function.

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