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FEKETE–SZEGÖ TYPE INEQUALITIES FOR CLASSES OF ANALYTIC FUNCTIONS DEFINED BY USING THE MODIFIED DZIOK–SRIVASTAVA AND THE OWA–SRIVASTAVA FRACTIONAL CALCULUS OPERATORS

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Abstract. By making use of the operator $\mathcal{K}_{\lambda_1,\lambda_2}^{m,r,s} f(z)$ which was previously defined as a generalization of Dziok-Srivastava operator [19, 17], the new class $S^*(\phi,m,r,s,\lambda_1,\lambda_2)$ was introduced and sharp upper bounds of $|a_3-\mu a_2^2|$ for the functions belonging to it were determined. Furthermore, Fekete-Szegö inequalities for certain classes of functions defined through fractional derivatives were also solved out in the sight of Owa-Srivastava fractional calculus operators.

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