THE t–MODIFICATION OF HOMOGENEOUS SYMMETRIC MEANS CONCERNING COMPLETE ELLIPTIC INTEGRALS

MIAO-KUN WANG* AND JIA-HUI WU

Abstract. In the article, we establish a monotonicity criterion for the t-modification of homogeneous symmetric mean, and provide several novel inequalities for certain homogeneous symmetric means concerning the complete elliptic integrals. Besides, we also give the answer to a conjecture recently proposed by Nishimura.

REFERENCES

[1] G. ALMKVIST, B. BERNDT, *Gauss, Landen, Ramanujan, the arithmetic-geometric mean, ellipse, π, and the Ladies Diary*, Amer. Math. Monthly, **95**, 7 (1988), 585–608.

[2] H. ALZER, S. L. QIU, *Monotonicity theorems and inequalities for the complete elliptic integrals*, J. Comput. Appl. Math., **172**, 2 (2004), 289–312.

[3] G. D. ANDERSON, S. L. QIU, M. K. VAMANAMURTHY, *Elliptic integral inequalities, with applications*, Constr. Approx., **14**, 2 (1998), 195–207.

[4] G. D. ANDERSON, M. K. VAMANAMURTHY, M. VUORINEN, *Distortion functions for plane quasiconformal mappings*, Israel J. Math., **62**, 1(1988), 1–16.

[5] G. D. ANDERSON, M. K. VAMANAMURTHY, M. VUORINEN, *Conformal Invariants, Inequalities, and Quasiconformal Maps*, John Wiley & Sons, New York, 1997.

[6] G. D. ANDERSON, M. K. VAMANAMURTHY, M. VUORINEN, *Topics in special functions*, in: Papers on Analysis, 5–26, Rep. Univ. Jyväskylä Dep. Math. Stat., **83**, Univ. Jyväskylä, Jyväskylä, 2001.

[7] R. W. BARNARD, K. PEARCE, K. C. RICHARDS, *An inequality involving the generalized hypergeometric function and the arc length of an ellipse*, SIAM J. Math. Anal., **31**, 3 (2000), 693–699.

[8] R. W. BARNARD, K. PEARCE, K. C. RICHARDS, *A monotonicity property involving $_3F_2$ and comparisons of the classical approximations of elliptical arc length*, SIAM J. Math. Anal., **32**, 2 (2000), 403–419.

[9] J. M. BORWEIN, P. B. BORWEIN, *Pi and the AGM*, John Wiley & Sons, New York, 1987.

[10] P. F. BYRD, M. D. FRIEDMAN, *Handbook of Elliptic Integrals of Engineer and Physicists*, Springer, New York, 1971.

[11] Y. M. CHU, M. K. WANG, *Optimal Lehmer Mean Bounds for the Toader Mean*, Results in Mathematics, **61**, 3–4 (2012), 223–229.

[12] Y. M. CHU, M. K. WANG, S. L. QIU, *Optimal combinations bounds of root-square and arithmetic means for Toader mean*, Proc. Indian Acad. Sci. Math. Sci., **122**, 1 (2012), 41–51.

[13] P. DUREN, J. PFALTZGRAFF, *Robin Capacity and Extremal Length*, J. Math. Anal. Appl., **179**, 1 (1993), 110–119.

[14] A. G. GREENHILL, *The Applications of Elliptic Functions*, Dover, New York, 1959.

[15] D. F. LAWDEN, *Elliptic Functions and Applications*, Applied Mathematics Sciences **80**, Springer-Verlag, New York, 1989.

[16] R. NISHIMURA, *Discrete monotonicity of means and its applications*, Proc. Amer. Math. Soc., **146**, 5 (2018), 2091–2102.

[17] O. LEHTO AND K. I. VIRTANEN, *Quasiconformal Mappings in the Plane*, Springer-Verlag, New York-Heidelberg, 1973.

[18] F. W. J. OLVER, D. W. LOZIER, R. F. BOISVERT, C. W. CLARK (Eds), *NIST Handbook of Mathematical Functions*, Cambridge Univ. Press, Cambridge, 2010.

[19] Y. F. QIU, *Properties of complete elliptic integrals and inequalities of means with applications*, Thesis (Master Degree), Zhejiang Sci-Tech University, 2011, 49 pages.

[20] S. TAKEUCHI, *A new form of the generalized complete elliptic integrals*, Kodai Math. J., **39**, 1 (2016), 202–226.

[21] GH. TOADER, *Some mean values related to the arithmetic-geometric mean*, J. Math. Anal. Appl., **218**, 2 (1998), 358–368.

[22] M. K. VAMANAMURTHY, M. VUORINEN, *Inequalities for means*, J. Math. Anal. Appl., **183**, 1 (1994), 155–166.

[23] M. K. WANG, Y. M. CHU, *Asymptotical bounds for complete elliptic integrals of the second kind*, J. Math. Anal. Appl., **402**, 1 (2013), 119–126.

[24] M. K. WANG, Y. M. CHU, S. L. QIU, Y. P. JIANG, *Convexity of the complete elliptic integrals of the first kind with respect to Hölder means*, J. Math. Anal. Appl., **388**, 2 (2012), 1141–1146.

[25] M. K. WANG, Y. M. CHU, S. L. QIU, Y. P. JIANG, *Bounds for the perimeter of an ellipse*, J. Approx. Theory, **164**, 7 (2012), 928–937.

[26] T.-H. ZHAO, M.-K. WANG, Y.-M. CHU, *On the bounds of the perimeter of an ellipse*, Acta Math. Sci., **42B**, 2 (2022), 491–501.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com