

## GENERALIZED ELLIPTIC INTEGRALS AND GENERALIZED GRÖTZSCH FUNCTION WITH TWO PARAMETERS

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**Abstract.** In this paper, we mainly study some monotonicity properties for the generalized  $(p, q)$ -elliptic integrals and the generalized Grötzsch function. As applications, we obtain some sharp functional inequalities for the generalized Grötzsch function and thus get improvements and extensions of some previous results.

**Mathematics subject classification (2020):** 33C05, 33E05, 33B10.

**Keywords and phrases:** Generalized elliptic integrals, generalized Grötzsch function, monotonicity, sharp inequalities.

### REFERENCES

- [1] H. ALZER, S. L. QIU, *Monotonicity theorems and inequalities for the complete elliptic integrals*, J. Comput. Appl. Math., 2004, 172 (2): 289–312.
- [2] G. D. ANDERSON, S. L. QIU, M. K. VAMANAMURTHY, M. VUORINEN, *Generalized elliptic integrals and modular equations*, Pac. J. Math., 2000, 192: 1–37.
- [3] G. D. ANDERSON, M. K. VAMANAMURTHY, M. VUORINEN, *Conformal Invariants, Inequalities, and Quasiconformal Maps*, John Wiley and Sons, New York, 1997, 48–73.
- [4] G. D. ANDERSON, M. K. VAMANAMURTHY, M. VUORINEN, *Functional inequalities for complete elliptic integrals and their ratios*, SIAM J. Math. Anal., 2012, 21 (2): 536–549.
- [5] B. C. BERNDT, *Ramanujan's Notebooks III*, Springer-Verlag, New York, 1985.
- [6] J. M. BORWEIN, P. B. BORWEIN, *Pi and the AGM*, A Wiley-Interscience Publication, New York, 1987, 1–32.
- [7] P. DRÁBEK, R. MANRÁSEVICH, *On the closed solution to some  $p$ -Laplacian nonhomogeneous eigenvalue problems*, Diff. Integral Equations, 1999, 12 (6): 733–788.
- [8] X. Y. GU, X. H. ZHANG, *Functional inequalities for the ratio of complete  $p$ -elliptic integrals*, J. Math. Anal. Appl., 2020, 488: 124065.
- [9] R. B. JIAO, S. L. QIU, G. T. GE, *Monotonicity and convexity properties of the generalized  $(p, q)$ -elliptic integrals*, J. Zhejiang Sci-Tech Univ., 2018, 39 (6): 765–769.
- [10] T. KAMIYA, S. TAKEUCHI, *Complete  $(p, q)$ -elliptic integrals with application to a family of means*, J. Classical Anal., 2017, 10 (1): 15–25.
- [11] O. LEHTO, K. I. VIRTANEN, *Quasiconformal Mappings in the Plane*, Springer-Verlag, New York, 1973.
- [12] X. Y. MA, S. L. QIU, G. Y. TU, *Generalized Grötzsch ring function and generalized elliptic integrals*, Appl. Math. A J. Chinese Univ., 2016, 31 (4): 458–468.
- [13] S. PONNUSAMY, M. VUORINEN, *Asymptotic expansion and inequalities for hypergeometric function*, Mathematika, 1997, 44 (2): 278–301.
- [14] S. L. QIU, Z. S. DING, J. WANG, *Some properties of the  $(p, q)$ -Grötzsch ring function and  $(p, q)$ -Hübner function*, J. Zhejiang Sci-Tech Univ., 2020, 43 (6): 846–851.
- [15] S. L. QIU, M. K. VAMANAMURTHY, M. VUORINEN, *Some inequalities for the growth of elliptic integrals*, SIAM J. Math. Anal., 2006, 29 (5): 1224–1237.
- [16] S. TAKEUCHI, *Generalized Jacobian elliptic functions and their application to bifurcation problems with  $p$ -Laplacian*, J. Math. Anal. Appl., 2012, 385 (1): 24–35.

- [17] S. TAKEUCHI, *Legendre-type relations for generalized complete elliptic integrals*, J. Classical Anal., 2016, 9 (1): 35–42.
- [18] S. TAKEUCHI, *A new form of the generalized complete elliptic integrals*, Kodai Math. J., 2016, 39 (1): 202–226.
- [19] F. WANG, J. H. HE, Y. LI, AND F. QI, *Monotonicity properties and inequalities related to generalized Grötzsch ring functions*, Open Math., 2019, 17 (1): 802–812.
- [20] X. H. ZHANG, *Solution to a conjecture on the Legendre  $M$ -function with an application to the generalized modulus*, J. Math. Anal. Appl. 2015, 431: 1190–1196.
- [21] X. H. ZHANG, *Monotonicity and functional inequalities for the complete  $p$ -elliptic integrals*, J. Math. Anal. Appl., 2017, 453: 942–953.
- [22] X. H. ZHANG, *On generalized modulus Ramanujan J*, 2017, 43: 405–413.