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ESTIMATES FOR WEIGHTED HARDY–LITTLEWOOD AVERAGES AND THEIR COMMUTATORS ON MIXED CENTRAL MORREY SPACES

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Abstract. In this paper, we study the boundedness of the weighted Hardy-Littlewood average H_φ and its commutator H_φ^b on mixed central Morrey spaces. More precisely, we first obtain the sufficient and necessary condition for the boundedness of H_φ on the mixed central Morrey space $\dot{M}_{\vec{q}}^{p}(\mathbb{R}^n)$, and also obtain the sharp constant simultaneously. Then we give a characterization for the boundedness of the commutator formed by H_φ and a central bounded mean oscillation function b on $\dot{M}_{\vec{q}}^{p}(\mathbb{R}^n)$.

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REFERENCES

[1] A. BENEDEK AND R. PANZONE, *The space L^p, with mixed norm*, Duke Math. J. 28 (1961), no. 3, 301–324.
[2] C. CARTON-LEBRUN AND M. FOSSET, *Moyennes et quotients de Taylor dans BMO*, Bull. Soc. Roy. Sci. Liege, 53 (1984), 85–87.
[3] T. CHEN AND W. C. SUN, *Iterated weak and weak mixed-norm spaces with applications to geometric inequalities*, J. Geom. Anal. 30 (2020), no. 4, 4268–4323.
[4] J. Y. CHU, Z. W. FU, AND Q. Y. WU, *L^p and BMO bounds for weighted Hardy operators on the Heisenberg group*, J. Inequal. Appl. 2016 (2016), 12 pages.
[5] N. M. CHUONG, D. V. DUONG, AND K. H. DUNG, *Multilinear Hausdorff operator on variable exponent Morrey-Herz type spaces*, Integral Transforms Spec. Funct. 31 (2020), no. 1, 62–86.
[6] N. M. CHUONG, N. T. HONG, AND H. D. HUNG, *Bounds of weighted multilinear Hardy-Cesàro operators in p-adic functional spaces*, Front. Math. China 13 (2018), no. 1, 1–24.
[7] N. M. CHUONG AND H. D. HUNG, *Bounds of weighted Hardy-Cesàro operators on weighted Lebesgue and BMO spaces*, Integral Transforms Spec. Funct. 25 (2014), no. 9, 697–710.
[8] N. M. CHUONG, D. VAN DUONG, AND K. H. DUNG, *Multilinear Hausdorff operators on some function spaces with variable exponent*, arXiv preprint arXiv:1709.08185, 2017.
[9] D. S. FAN AND F. Y. ZHAO, *Sharp constants for multilinear Hausdorff q-inequalities*, J. Aust. Math. Soc. 106 (2019), no. 2, 274–286.
[10] Z. W. FU, S. L. GONG, S. Z. LU, AND W. YUAN, *Weighted multilinear Hardy operators and commutators*, Forum Math. 27 (2015), no. 5, 2825–2851.
[11] Z. W. FU, Z. G. LIU, AND S. Z. LU, *Commutators of weighted Hardy operators on $\mathbb{R}^n$*, Pro. Amer. Math. Soc. 137 (2009), no. 10, 3319–3328.
[12] Z. W. FU, Z. G. LIU, S. Z. LU, AND H. B. WANG, *Characterization for commutators of n-dimensional fractional Hardy operators*, Sci. China Ser. A, 50 (2007), no. 10, 1418–1426.
[13] Z. W. FU AND S. Z. LU, *Weighted Hardy operators and commutators on Morrey spaces*, Front. Math. China, 5 (2010), no. 3, 531–539.
[14] G. L. GAO, X. M. WU, AND W. C. GUO, *Some results for Hausdorff operators*, Math. Inequal. Appl. 18 (2015), no. 1, 155–168.
[15] G. L. GAO, X. M. WU, A. HUSSAIN, AND G. P. ZHAO, *Some estimates of Hausdorff operators*, J. Math. Inequal. 9 (2015), 641–651.

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[16] G. L. GAO AND Y. ZHONG, *Some inequalities for Hausdorff operators*, Math. Inequal. Appl. 17 (2014), no. 3, 1061–1078.
[17] G. L. GAO AND Y. ZHONG, *Some estimates of Hardy operators and their commutators on Morrey-Herz spaces*, J. Math. Inequal. 11 (2017), no. 1, 49–58.
[18] L. GRAFAKOS, *Modern Fourier analysis*, vol. 250, Springer, 2009.
[19] K.-P. HO, *Mixed norm Lebesgue spaces with variable exponents and applications*, Riv. Mat. Univ. Parma, 9 (2018), no. 1, 21–44.
[20] H. D. HUNG, *The p-adic weighted Hardy-Cesàro operator and an application to discrete Hardy inequalities*, J. Math. Anal. Appl. 409 (2014), no. 2, 868–879.
[21] C. B. MORREY, *On the solutions of quasi-linear elliptic partial differential equations*, Tran. Amer. Math. Soc. 43 (1938), no. 1, 126–166.
[22] T. NOGAYAMA, *Boundedness of commutators of fractional integral operators on mixed Morrey spaces*, Integral Transforms Spec. Funct. 30 (2019), no. 10, 790–816.
[23] T. NOGAYAMA, *Mixed Morrey spaces*, Positivity, 23 (2019), no. 4, 961–1000.
[24] T. NOGAYAMA, T. ONO, D. SALIM, AND Y. SAWANO, *Atomic decomposition for mixed Morrey spaces*, J. Geom. Anal. 31 (2021), no. 9, 9338–9365.
[25] Y. SAWANO, G. DI FAZIO, AND D. I. HAKIM, *Morrey Spaces: Introduction and applications to integral operators and PDE's*, Volume I & II, CRC Press, 2020.
[26] C. Q. TANG, F. Y. XUE, AND Y. ZHOU, *Commutators of weighted Hardy operators on Herz-type spaces*, Ann. Pol. Math. 101 (2011), no. 3, 267–273.
[27] S. VOLOSIVETS, *Weighted Hardy and Cesàro operators on Heisenberg group and their norms*, Integral Transforms Spec. Funct. 28 (2017), no. 12, 940–952.
[28] M. Q. WEI, *A characterization of* $\dot{\mathrm{CMO}}^{\vec{q}}$ *via the commutator of Hardy type operators on mixed Herz spaces*, Appl. Anal. published online, 16 pages, 2021.
[29] X. M. WU, *Necessary and sufficient conditions for generalized Hausdorff operators and commutators*, Ann. Funct. Anal. 6 (2015), no. 3, 60–72.
[30] J. XIAO, *L^p and BMO bounds of weighted Hardy–Littlewood averages*, J. Math. Anal. Appl. 262 (2001), no. 2, 660–666.