

## SINGULAR VALUE INEQUALITIES FOR ACCRETIVE–DISSIPATIVE NORMAL OPERATORS

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**Abstract.** In this paper, we present singular value inequalities relevant to accretive-dissipative normal compact operators. In particular, we showed that if  $X = A + iB$  and  $Y = C + iD$  are accretive-dissipative normal compact operators, where  $A \leq C$  and  $B \leq D$ . Then

$$s_j(X - Y) \leq \sqrt{2}s_j(X \oplus Y)$$

for  $j = 1, 2, \dots$ . Moreover, if  $\begin{bmatrix} X & Z \\ Z^* & Y \end{bmatrix}$  is accretive-dissipative normal compact operator. Then

$$\sqrt{2}s_j(Z) \leq s_j(X \oplus Y)$$

for  $j = 1, 2, \dots$ . We showed that these inequalities are equivalent. Also, we provide several singular value inequalities relevant to accretive-dissipative normal compact operators.

**Mathematics subject classification (2020):** 15A18, 15A42, 47A63, 47B07, 47B15.

**Keywords and phrases:** Singular value, compact operator, accretive-dissipative operator, normal operator.

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