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A NEW SUBCLASS OF CLOSE–TO–CONVEX HARMONIC MAPPINGS

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Abstract. In this paper, we introduce and investigate a new subclass of harmonic mappings which satisfy a third-order differential inequality. Such results as close-to-convexity, coefficient bounds, growth estimates, sufficient coefficient condition and convolution properties are derived. Furthermore, we obtain several improved versions of the sharp Bohr radius for harmonic mappings.

Mathematics subject classification (2020): Primary 30C45, 30C80.
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