

MULTILINEAR COMMUTATORS RELATED TO MAXIMAL FUNCTION ON MORREY–BANACH SPACE AND ITS APPLICATION

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Abstract. The authors get the equivalent conditions for the boundedness of the commutators generated by the multilinear maximal functions and the BMO functions on Morrey-Banach space. As applications, we obtain the equivalent conditions for the boundedness of such operators on Morrey spaces with variable exponents and Morrey-Lorentz space which are all new results in the multi-linear case. Moreover, as far as we know, the results of this paper seem to be new even for the one-linear case.

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