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ON MONOTONIC L_φ–SOLUTIONS FOR A CLASS OF QUADRATIC–URYSOHN INTEGRAL EQUATIONS

MOHAMED M. A. METWALI

Abstract. This article is devoted to study the existence of a.e. monotonic solutions of functional quadratic Urysohn integral equations in Orlicz spaces L_φ. Due to various continuity properties of the operators in Orlicz spaces, there are many different cases to discuss the considered problem. We focus on assumptions permitting us to consider strongly nonlinear operators and to combine the results of both standard and quadratic integral equations. We discuss the studied problem in three general and different cases when the function φ satisfies Δ', Δ_2, and Δ_3-conditions separately under a general set of assumptions.

Mathematics subject classification (2020): 45G10, 47H30, 47N20, 46E30.
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REFERENCES

[1] R. P. AGARWAL, D. O'REGAN, AND P. WONG, *Constant-sign solutions of a system of Volterra integral equations in Orlicz spaces*, J. Integral Equations Appl., **20**, (2008), 337–378.
[2] I. K. ARGYROS, *Quadratic equations and applications to Chandrasekhar and related equations*, Bull. Austral. Math. Soc., **32**, (1985), 275–292.
[3] J. BANAŚ AND K. GOEBEL, *Measures of Noncompactness in Banach Spaces*, Lect. Notes in Math., **60**, M. Dekker, New York–Basel, 1980.
[4] J. BANAŚ AND Z. KNAP, *Integrable solutions of a functional-integral equation*, Rev. Mat. Univ. Complut. Madrid, **2**, (1989), 31–38.
[5] J. BANAŚ, A. MARTINON, *Monotonic solutions of a quadratic integral equation of Volterra type*, Comp. Math. Appl., **47**, (2004), 271–279.
[6] J. BANAŚ AND L. OLSZOWY, *Measures of noncompactness related to monotonicity*, Comment. Math. Prace Mat, **41**, (2001), 13–23.
[7] C. BARDARO, J. MUSIELAK AND G. VINTI, *Nonlinear Integral Operators and Applications*, Walter de Gruyter, Berlin, New York, 2003.
[8] A. BELLOUR, D. O'REGAN, AND M.-A. TAOUDI, *On the existence of integrable solutions for a nonlinear quadratic integral equation*, J. Appl. Math. Comput., **46**, (2013), 67–77.
[9] J. CABALLERO, A. B. MINGARELLI, AND K. SADARANGANI, *Existence of solutions of an integral equation of Chandrasekhar type in the theory of radiative transfer*, Electr. Jour. Differ. Equat., **57**, (2006), 1–11.
[10] S. CHANDRASEKHAR, *Radiative Transfer*, Dover Publications, New York, 1960.
[11] I.-Y. S. CHENG AND J. J. KOZAK, *Application of the theory of Orlicz spaces to statistical mechanics. I. Integral equations*, J. Math. Phys., **13**, (1972), 51–58.
[12] M. CICHOŃ AND M. METWALI, *Existence of monotonic L_ϕ-solutions for quadratic Volterra functionl integral equations*, Electron. J. Qual. Theory Differ. Equ., **13**, (2015), 1–16.
[13] M. CICHOŃ AND M. METWALI, *On a fixed point theorem for the product of operators*, Jour. Fixed Point Theory Appl., **18**, (2016), 753–770.
[14] M. CICHOŃ AND M. METWALI, *On quadratic integral equations in Orlicz spaces*, J. Math. Anal. Appl., **387**, (2012), 419–432.

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[15] M. CICHOŃ AND M. METWALI, *On solutions of quadratic integral equations in Orlicz spaces*, Mediterr. J. Math., **12**, (2015), 901–920.
[16] M. CICHOŃ AND M. METWALI, *On the existence of solutions for quadratic integral equations in Orlicz space*, Math. Slovaca, **66**, (2016), 1413–1426.
[17] G. EMMANUELE, *About the existence of integrable solutions of functional-integral equation*, Rev. Mat. Univ. Complut. Madrid, **4**, (1991), 65–69.
[18] N. ERZAKOVA, *Compactness in measure and measure of noncompactness*, Siberian Math. J., **38**, (1997), 926–928.
[19] H. HASHEM AND A. EL-SAYED, *Stabilization of coupled systems of quadratic integral equations of Chandrasekhar type*, Math. Nachr., **290**, (2017), 341–348.
[20] M. A. KRASNOSEL'SKII AND YU. RUTITSKII, *Convex Functions and Orlicz Spaces*, Gröningen, 1961.
[21] M. KUNZE, *On a special class of nonlinear integral equations*, J. Integral Equ. Appl., **7**, (1995), 329–350.
[22] L. MALIGRANDA, *Orlicz spaces and interpolation*, Campinas SP Brazil: Departamento de Matemática, Universidade Estadual de Campinas, 1989.
[23] M. METWALI, *On a class of quadratic Urysohn-Hammerstein integral equations of mixed type and initial value problem of fractional order*, Mediterr. J. Math., **13**, (2016), 2691–2707.
[24] M. METWALI, *On perturbed quadratic integral equations and initial value problem with nonlocal conditions in Orlicz spaces*, Demonstratio Mathematica, **53**, (2020), 86–94.
[25] M. METWALI, *Solvability of functional quadratic integral equations with perturbation*, Opuscula Math., **33**, 4 (2013), 725–739.
[26] D. O'REGAN, *Solutions in Orlicz spaces to Urysohn integral equations*, Proceedings of the Royal Irish Academy, Section A, **96**, (1996), 67–78.
[27] W. ORLICZ AND S. SZUFLA, *On some classes of nonlinear Volterra integral equations in Banach spaces*, Bull. Acad. Polon. Sci. Sér. Sci. Math., **30**, (1982), 239–250.
[28] R. PŁUCIENNIK AND S. SZUFLAPERU, *Nonlinear Volterra integral equations in Orlicz spaces*, Demonstratio Math., **17**, (1984), 515–532.
[29] M. M. RAO AND Z. D. REN, *Theory of Orlicz Spaces*, Marcel Dekker, New York, 1991.
[30] A. SOŁTYSIAK AND S. SZUFLA, *Existence theorems for L_φ-solutions of the Hammerstein integral equation in Banach spaces*, Comment. Math. Prace Mat., **30**, (1990), 177–190.
[31] M. VÄTH, *Volterra and Integral Equations of Vector Functions*, Marcel Dekker, New York-Basel, 2000.
[32] J. D. WEEKS, S. A. RICE, AND J. J. KOZAK, *Analytic approach to the theory of phase transitions*, J. Chem. Phys., **52**, (1970), 2416–2426.