

## HERMITE–HADAMARD TYPE INTEGRAL INEQUALITIES FOR THE CLASS OF STRONGLY CONVEX FUNCTIONS ON TIME SCALES

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**Abstract.** In this paper, we introduce the notion of a strongly convex function with respect to two non-negative auxiliary functions on time scales. We establish several new dynamic inequalities for these classes of strongly convex functions. The results obtained in this paper are the generalization of the results of Rashid *et al.* (Mathematics, 7 (10), 956, 2019). Further, we discuss some special cases which may be deduced from our main results. Moreover, some examples of our main results are mentioned.

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