GENERALIZED INEQUALITIES FOR FUNCTIONS OF L_p SPACES VIA MONTGOMERY IDENTITY WITH PARAMETERS

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Abstract. We obtain new Montgomery identity with parameters for two variables. By using this obtained identity, we give some generalization of Ostrowski type inequality for L_p spaces with better bounds. In addition, we modify Grüss type inequality for two independent variables involving parameters.

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