POST–QUANTUM OSTROWSKI TYPE INTEGRAL INEQUALITIES FOR TWICE (p,q)–DIFFERENTIABLE FUNCTIONS

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Abstract. In this paper, we establish a new (p,q)-integral identity using twice (p,q)-differentiable functions. Then, we use this result to derive some new post-quantum Ostrowski type integral inequalities for twice (p,q)-differentiable functions. The newly established results are also proven to be generalizations of some existing results in the area of integral inequalities.

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REFERENCES

[1] M. A. ALI, M. ABBAS, H. BUDAK, P. AGARWAL, G. MURTAZA, Y. M. CHU, *Quantum variant of Montgomery identity and Ostrowski-type inequalities for the mappings of two variables*, Adv. Differ. Equ., **2021**, 2021, 25.

[2] M. A. ALI, H. BUDAK, M. ABBAS, M. Y. CHU, *Quantum Hermite-Hadamard-type inequalities for functions with convex absolute values of second q^b-derivatives*, Adv. Differ. Equ., **2021**, 2021, 7.

[3] M. A. ALI, H. BUDAK, A. AKKURT, Y. M. CHU, *Quantum Ostrowski-type inequalities for twice quantum differentiable functions in quantum calculus*, Open Math. **19**, 2021, 440–449.

[4] M. A. ALI, S. K. NTOUYAS, J. TARIBOON, *Generalization of quantum Ostrowski-type integral inequalities*, Mathematics., **9 (10)**, 2021, 1155.

[5] A. A. ALJINOVIĆ, D. KOVAČEVIĆ, M. KUNT, M. PULJIZ, *Correction: Quantum Montgomery identity and quantum estimates of Ostrowski type inequalities*, AIMS Math., **6 (2)**, 2021, 1880–1888.

[6] A. A. ALJINOVIĆ, D. KOVAČEVIĆ, M. PULJIZ, A. Ž. KEKO, *On Ostrowski inequality for quantum calculus*, Appl. Math. Comput., **410**, 2021, 126454.

[7] M. W. ALOMARI, *A generalization of weighted companion of Ostrowski integral inequality for mappings of bounded variation*, RGMIA Res. Rep. Collect., **14 (87)**, 2011, 11.

[8] W. AL-SALAM, *Some fractional q-integrals and q-derivatives*, Proc. Edinburgh Math. Soc., **15 (2)**, 1966, 135-140.

[9] S. ARACI, U. DURAN, M. ACIKGOZ, H. M. SRIVASTAVA, *A certain (p,q)-derivative operator and associated divided differences*, J. Inequal. Appl., **2016**, 2016, 301.

[10] A. ARAL, V. GUPTA, R. P. AGARWAL, *Applications of q-calculus in operator theory*, Springer: Science+Business Media, 2013.

[11] S. ASAWASAMRIT, C. SUDPRASERT, S. NTOUYAS, J. TARIBOON, *Some result on quantum Hanh integral inequalities*, J. Inequal. Appl., **2019**, 2019, 154.

[12] G. BANGEREZAKO, *Variational q-calculus*, J. Math. Anal. Appl., **289**, 2004, 650–665.

[13] S. BERMUDO, P. KÓRUS, J. E. NÁPOLES VALDÉS, *On q-Hermite-Hadamard inequalities for general convex functions*, Acta Math. Hungar., **162**, 2020, 364–374.

[14] H. BUDAK, S. ERDEN, M. A. ALI, *Simpson and Newton-type inequalities for convex functions via newly defined quantum integrals*, Math. Meth. Appl. Sci., **44**, 2020, 378–390.

[15] H. BUDAK, M. A. ALI, N. ALP, Y. M. CHU, *Quantum Ostrowski type integral inequalities*, J. Math. Inequal., 2021.

[16] H. BUDAK, M. A. ALI, M. TARHANACI, *Some new quantum Hermite-Hadamard-like inequalities for coordinated convex functions*, J. Opt. Theory Appl., **186**, 2020, 899–910.

[17] H. BUDAK, M. A. ALI, T. TUNÇ, *Quantum Ostrowski-type integral inequalities for functions of two variables*, Math. Meth. Appl. Sci., **44**, 2021, 5857–5872.

[18] H. BUDAK, M. Z. SARIKAYA, *A companion of Ostrowski type inequalities for mappings of bounded variation and some applications*, Trans. A. Razmadze Math. Inst., **171 (2)**, 2017, 136–143.

[19] H. BUDAK, M. Z. SARIKAYA, *A new Ostrowski type inequality for functions whose first derivatives are of bounded variation*, Moroccan J. Pure Appl. Anal., **2 (1)**, 2016, 1–11.

[20] H. BUDAK, M. Z. SARIKAYA, *New weighted Ostrowski type inequalities for mappings with first derivatives of bounded variation*, Transylvanian J. Math. Mech., **8 (1)**, 2016, 21–27.

[21] P. CERONE, S. S. DRAGOMIR, J. ROUMELIOTIS, *An inequality of Ostrowski-type for mappings whose second derivatives are bounded and applications*, East Asian Math. J., **15**, 1999, 1–9.

[22] R. CHAKRABARTI, R. A. JAGANNATHAN, *(p,q)-oscillator realization of two-paramenter quantum algebras*, J. Phys. A Math. Gen., **24**, 1991, L711–L718.

[23] H. H. CHU, H. KALSOOM, S. RASHID, M. IDREES, F. SAFDAR, Y. M. CHU, D. BALEANU, *Quantum analogs of Ostrowski-type inequalities for raina's function correlated with coordinated generalized ϕ-convex functions*, Symmetry, **12 (2)**, 2020, 308.

[24] A. DOBROGOWSKA, A. ODZIJEWICZ, *Second order q-difference equation solvable by factorization method*, J. Comput. Appl. Math., **193**, 2006, 319–346.

[25] S. S. DRAGOMIR, *A companion of Ostrowski's inequality for functions of bounded variation and applications*, Int. J. Nonlinear Anal. Appl. **5 (1)**, 2014, 89–97.

[26] S. S. DRAGOMIR, *The Ostrowski's integral inequality for Lipschitzian mappings and applications*, Comput. Math. Appl., **38**, 1999, 33–37.

[27] U. DURAN, M. ACIKGOZ, A. ESI, S. ARACI, *A note on the (p,q)-Hermite polynomials*, Appl. Math. Inf. Sci., **12**, 2018, 227–231.

[28] T. ERNST, *A method for q-calculus*, J. Nonlinear Math. Phys., **10**, 2003, 487–525.

[29] H. EXTON, *q-Hypergeometric functions and applications*, New York: Halstead Press, 1983.

[30] H. GAUCHMAN, *Integral inequalities in q-calculus*, J. Comput. Appl. Math., **47**, 2002, 281–300.

[31] A. M. GAVRILIK, *q-serre relations in $U_q(u_n)$ and q-deformed meson mass sum rules*, J. Phys. A: Math. Gen., **27**, 1994, L91–L94.

[32] M. E. H. ISMAIL, P. SIMEONOV, *q-difference operators for orthogonal polynomials*, J. Comput. Appl. Math., **233**, 2009, 749–761.

[33] F. H. JACKSON, *On a q-definite integrals*, Quart. J. Pure Appl. Math., **41**, 1910, 193–203.

[34] S. JHANTHANAM, J. TARIBOON, S. K. NTOUYAS, K. NONLAOPON, *On q-Hermite-Hadamard inequalities for differentiable convex functions*, Mathematics., **7**, 2019, 632.

[35] V. KAC, P. CHEUNG, *Quantum Calculus*, Springer: New York, NY, USA, 2002.

[36] H. KALSOOM, M. AMER, M.-U.-D. JUNJUA, S. HUSSAIN, G. SHAHZADI, *Some (p,q)-estimates of Hermite-Hadamard-type inequalities for coordinated convex and quasi-convex functions*, Mathematics., **7**, 2019, 683.

[37] M. KUNT, A. KASHURI, T. DU, A. W. BAIDAR, *Quantum Montgomery identity and quantum estimates of Ostrowski type inequalities*, AIMS Math. **5 (6)**, 2020, 5439–5457.

[38] M. A. LATIF, S. HUSSAIN, *New inequalities of Ostrowski type for co-ordinated convex functions via fractional integrals*, J. Fractional Calc. Appl. **2 (9)**, 2012, 1–15.

[39] Z. LIU, *Some companions of an Ostrowski type inequality and applications*, J. Inequal. Pure Appl. Math. **10 (2)**, 2009, 52.

[40] W. LUANGBOON, K. NONLAOPON, J. TARIBOON, S. K. NTOUYAS, *On Simpson type inequalities for generalized strongly preinvex functions via (p,q)-calculus and applications*, AIMS Math., **6 (9)**, 2021, 9236–9261.

[41] W. LUANGBOON, K. NONLAOPON, J. TARIBOON, S. K. NTOUYAS, *Simpson- and Newton-type inequalities for convex functions via (p,q)-calculus*, Mathematics., **9**, 2021, 1338.

[42] B. MEFTAH, K. BOUKERRIOUA, *Some new Ostrowski-type inequalities for functions whose second derivative are h-convex via Riemann-Liouville fractional*, Malaya J. Matematik., **2 (4)**, 2014, 445–459.

[43] M. NASIRUZZAMAN, A. MUKHEIMER, M. MURSALEEN, *Some opial-type integral inequalities via (p,q)-calculus*, J. Inequal. Appl., **2019**, 2019, 295.

[44] M. A. NOOR, M. U. AWAN, K. I. NOOR, *Quantum Ostrowski inequalities for q-differentiable convex functions*, J. Math. Inequal., **10 (4)**, 2016, 1013–1018.

[45] A. M. OSTROWSKI, *Über die absolutabweichung einer differentiebaren funktion vonihrem integralmitelwert*, Comment. Math. Helv., **10**, 1938, 226–227.

[46] D. N. PAGE, *Information in black hole radiation*, Phys. Rev. Lett., **71**, 1993, 3743–3746.

[47] J. PRABSEANG, K. NONLAOPON, J. TARIBOON, *(p,q)-Hermite-Hadamard inequalities for double integral and (p,q)-differentiablen convex functions*, Axioms., **8**, 2019, 68.

[48] J. PRABSEANG, K. NONLAOPON, J. TARIBOON, *Quantum Hermite-Hadamard inequalities for double integral and q-differentiable convex functions*, J. Math. Inequal., **13**, 2019, 675–686.

[49] P. P. RAYCHEV, R. P. ROUSSEV, Y. F. SMIRNOV, *The quantum algebra $SU_q(2)$ and rotational spectra of deformed nuclei*, J. Phys. G: Nucl. Part. Phys., **16**, 1990, 137–141.

[50] P. N. SADJANG, *On the fundamental theorem of (p,q)-calculus and some (p,q)-Taylor formulas*, Results Math., **73**, 2018, 1–21.

[51] M. Z. SARIKAYA, H. BUDAK, *Generalized Ostrowski type inequalities for local fractional integrals*, Proc. Amer. Math. Soc. **145**, 2017, 1527–1538.

[52] E. SET, *New inequalities of Ostrowski type for mappings whose derivatives are s-convex in the second sense via fractional integrals*, Comput. Math. Appl., **63 (7)**, 2012, 1147–1154.

[53] J. SOONTHARANON, T. SITTHIWIRATTHAM, *Fractional (p,q)-calculus*, Adv. Differ. Equ., **2020**, 2020, 35.

[54] W. SUDSUTAD, S. K. NTOUYAS, J. TARIBOON, *Quantum integral inequalities for convex functions*, J. Math. Inequal., **9**, 2015, 781–793.

[55] J. TARIBOON, S. K. NTOUYAS, *Quantum calculus on finite intervals and applications to impulsive difference equations*, Adv. Differ. Equ., **2013**, 2013, 282.

[56] J. TARIBOON, S. K. NTOUYAS, *Quantum integral inequalities on finite intervals*, J. Inequal. Appl., **2014**, 2014, 121.

[57] M. TUNÇ, E. GÖV, *(p,q)-Integral inequalities*, RGMIA Res. Rep. Coll., **19 (97)**, 2016, 1–13.

[58] M. TUNÇ, E. GÖV, *Some integral inequalities via (p,q)-calculus on finite intervals*, RGMIA Res. Rep. Coll., **19 (95)**, 2016, 1–12.

[59] M. VIVAS-CORTEZ, M. A. ALI, H. BUDAK, H. KALSOOM, P. AGARWAL, *Some new Hermite-Hadamard and related inequalities for convex functions via (p,q)-Integral*, Entropy., **23 (7)**, 2021, 828.

[60] W. YANG, *Some new Fejér type inequalities via quantum calculus on finite intervals*, Science Asia., **43**, 2017, 123–134.

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