STRONG DEVIATION THEOREMS FOR
GENERAL INFORMATION SOURCES

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Abstract. In this paper, we first introduce some new concepts of generalized likelihood ratio, upper/lower generalized divergence rate and upper/lower generalized relative entropy, as a measure of randomness to characterize the deviation between generalized information sources and memoryless (i.e., independent) sources. Then, by adopting pure analysis method on studying probability limit theory, a class of strong limit theorems and strong deviation theorems for generalized information sources and generalized information source entropy density are established. The outcomes extend some existing results of [10] and [21].

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