

APPROXIMATION PROPERTIES OF THE RIEMANN–LIOUVILLE FRACTIONAL INTEGRAL TYPE SZÁSZ–MIRAKYAN–KANTOROVICH OPERATORS

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Abstract. In the present paper, we introduce the Riemann-Liouville fractional integral type Szász-Mirakyan-Kantorovich operators. We investigate the order of convergence by using Lipschitz-type maximal functions, second order modulus of smoothness and Peetre's K -functional. Weighted approximation properties of these operators in terms of modulus of continuity have been discussed. Then, Vorononskaja-type theorem are obtained. Moreover, bivariate the Riemann-Liouville fractional integral type Szász-Mirakyan-Kantorovich operators are constructed. The last section is devoted to graphical representation and numerical results for these operators.

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