

HERMITE–HADAMARD TYPE INEQUALITIES FOR RIEMANN–LIOUVILLE FRACTIONAL INTEGRALS VIA STRONGLY h -CONVEX FUNCTIONS

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Abstract. In this paper, the Hermite–Hadamard type inequalities for Riemann–Liouville fractional integrals via strongly h -convex functions are established. Furthermore, we obtain some identities related to the fractional integrals with n -times differentiable functions, and then gain midpoint type and trapezoid type error estimates connected with the Hermite–Hadamard type inequalities, which generalize some known results.

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