SOME GENERALIZATIONS ON q–STEFFENSEN INEQUALITY

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Abstract. In this work, we study Steffensen inequality and obtain some generalizations on q-analogue of Steffensen inequality for infinite sums without restricted to the bounds. Since there are some differences between quantum and classical calculus, such as q-integral of a positive function from a to b ($a, b \in \mathbb{R}^+$, $0 < a < b$, $0 < q < 1$) does not have to be positive. The obtained results can not be exactly expressed as classical ones, nonetheless to say, they are q-extensions of the results in the classical ones.

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