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A REFINED HARDY–LITTLEWOOD–POLYA INEQUALITY AND THE EQUIVALENT FORMS

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Abstract. In this article, by the Euler-Maclaurin summation formula, we construct proper weight coefficients and use them to establish a refined Hardy-Littlewood-Polya inequality with multi parameters. Based on this inequality, the equivalent statements of the best possible constant factor related to several parameters are discussed. The equivalent forms, some particular inequalities and the operator expressions of the obtained inequalities are considered.

Mathematics subject classification (2020): 26D15, 26D10, 47A05.
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