

A WEIGHTED QUANTITATIVE ISOPERIMETRIC INEQUALITY FOR KORÁNYI SPHERE IN HEISENBERG GROUP \mathbb{H}^n

GUOQING HE AND PEIBIAO ZHAO*

Abstract. It is well known that the Korányi sphere *w.r.t.* the Korányi distance is not an isoperimetric set in Heisenberg group \mathbb{H}^n . In this paper, we investigate Korányi sphere in a Heisenberg group associated with a density $|z|^{-(2n+1)}e^{-\frac{\alpha}{|z|}}$ ($\alpha > 0$), and derive a weighted isoperimetric inequality and a weighted quantitative isoperimetric inequality for Korányi spheres in half-cylinders. This note also shows that the Korányi sphere is the weighted isoperimetric set in the weighted Heisenberg group \mathbb{H}^n .

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