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MAXIMAL COMMUTATOR AND COMMUTATOR OF MAXIMAL FUNCTION ON TOTAL MORREY SPACES

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Abstract. In this paper we introduce a new variant of Morrey spaces called total Morrey spaces $L^{p,\lambda,\mu}(\mathbb{R}^n)$. These spaces generalize the classical Morrey spaces so that $L^{p,\lambda,\lambda}(\mathbb{R}^n) \equiv L^{p,\lambda}(\mathbb{R}^n)$ and the modified Morrey spaces so that $L^{p,\lambda,0}(\mathbb{R}^n) = \widetilde{L}^{p,\lambda}(\mathbb{R}^n)$. We give basic properties of the spaces $L^{p,\lambda,\lambda}(\mathbb{R}^n)$ and study some embeddings into the Morrey space $L^{p,\lambda,\mu}(\mathbb{R}^n)$. We also give necessary and sufficient conditions for the boundedness of the maximal commutator operator M_b and commutator of maximal operator $[b,M]$ on $L^{p,\lambda,\mu}(\mathbb{R}^n)$. We obtain some new characterizations for certain subclasses of $BMO(\mathbb{R}^n)$.

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