

SOME DOMINATION INEQUALITIES FOR SPECTRAL ZETA KERNELS ON CLOSED RIEMANNIAN MANIFOLDS

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Abstract. We first prove Kato's inequalities for the Laplacian and a Schrödinger-type operator on smooth functions on closed Riemannian manifolds. We then apply the result to establish some new domination inequalities for spectral zeta functions and their related spectral zeta kernels on n -dimensional unit spheres using Kato's inequalities and majorisation techniques. Our results are the generalisations of Kato's comparison inequalities for Riemannian surfaces to n -dimensional closed Riemannian manifolds.

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