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THE MOLECULAR DECOMPOSITION OF ANISOTROPIC MIXED–NORM HARDY SPACES AND THEIR APPLICATION

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Abstract. Let $\vec{p} \in (0,\infty)^n$, A be an expansive dilation on $\mathbb{R}^n$, and $H_A^{\vec{p}}(\mathbb{R}^n)$ be the anisotropic mixed-norm Hardy space defined via the non-tangential grand maximal function. In this paper, the authors establish its molecular decomposition. As an application, the authors obtain the boundedness of a class of singular integral operators from $H_A^{\vec{p}}(\mathbb{R}^n)$ to $H_A^{\vec{p}}(\mathbb{R}^n)$. These results are still new even in the classical isotropic setting (in the case $A := 2\mathrm{I}_{n\times n}$).
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