Journal of Mathematical Inequalities
Volume 17, Number 1 (2023), 67–82
doi:10.7153/jmi-2023-17-05

LYAPUNOV–TYPE INEQUALITIES FOR FRACTIONAL LANGEVIN DIFFERENTIAL EQUATIONS

ZAID LAADJAL AND QINGHUA MA*

Abstract. In this paper, we establish some new Lyapunov-type inequalities for Langevin equations involving derivatives of fractional orders with two classes of two-point boundary conditions, which have generalized some previous results.

Mathematics subject classification (2020): 34A08, 26A33, 26D10, 34B27.
Keywords and phrases: Lyapunov-type inequality, Liouville-Caputo fractional derivative, fractional Langevin equation, Green's function.

REFERENCES

[1] T. ABDELJAWAD, F. JARAD, S. F. MALLAK, J. ALZABUT, *Lyapunov type inequalities via fractional proportional derivatives and application on the free zero disc of Kilbas-Saigo generalized Mittag-Leffler functions*, Eur. Phys. J. Plus, **134**, 247 (2019), 1–14.

[2] B. AHMAD, J. NIETO, *Solvability of nonlinear Langevin equation involving two fractional orders with Dirichlet boundary conditions*, Int. J. Differ. Equ., ID 1649486 (2010), 1–10.

[3] B. AHMAD, J. J. NIETO, A. ALSAEDI, M. EL-SHAHED, *A study of nonlinear Langevin equation involving two fractional orders in different intervals*, Nonlinear Analys. Real World Appl., **13**, 2 (2012), 599–606.

[4] S. BUROV, E. BARKAI, *Critical exponent of the fractional Langevin equation*, Phys. Rev. Lett., **100**, ID 070601 (2008), 1–4.

[5] D. CAKMAK, *Lyapunov-type integral inequalities for certain higher order differential equations*, Appl. Math. Comput., **216**, 2 (2010), 368–373.

[6] S. I. DENISOV, H. KANTZ, AND P. HANGGI, *Langevin equationwith super-heavy-tailed noise*, J. Phys. A, **43**, 28, ID 285004 (2010).

[7] K. DIETHELM, *The Analysis of Fractional Differential Equations: An Application-Oriented Exposition Using Differential Operators of Caputo Type*, Springer, Berlin, 2010.

[8] S. B. ELIASON, *Lyapunov type inequalities for certain second order functional differential equations*, SIAM J. Appl. Math., **27**, 1 (1974), 180–199.

[9] R. A. C. FERREIRA, *A Lyapunov-type inequality for a fractional boundary value problem*, Fract. Calc. Appl. Anal., **16**, 4 (2013), 978–984.

[10] R. A. C. FERREIRA, *Lyapunov-type inequalities for some sequential fractional boundary value problems*, Adv. Dyn. Syst. Appl., **11**, 1 (2016), 33–43.

[11] R. A. C. FERREIRA, *Novel Lyapunov-type inequalities for sequential fractional boundary value problems*, Rev. R. Acad. Cienc. Exactas Fis. Nat. Ser. A Math., **113**, 1 (2019), 171–179.

[12] R. A. C. FERREIRA, *On a Lyapunov-type in equality and the zeros of a certain Mittag–Leffler function*, J. Math. Anal. Appl., **412**, 2 (2014), 1058–1063.

[13] S. HARIKRISHNAN, K. KANAGARAJAN, E. M. ELSAYED, *Existence and stability results for Langevin equations with Hilfer fractional derivative*, Res. Fixed Point Theory Appl., ID 20183, (2018), 1–10.

Paper JMI-17-05

[14] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRUJILLO, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, **204**, Elsevier Science B. V, Amsterdam, 2006.

[15] Z. LAADJAL, B. AHMED, N. ADJEROUD, *Existence and uniqueness of solutions for multiterm fractional Langevin equation with boundary conditions*, DCDIS Series A: Math. Analys., **27**, 5a (2020), 339–350.

[16] Z. LAADJAL, F. JARAD, *Existence, uniqueness and stability of solutions for generalized proportional fractional hybrid integro-differential equations with Dirichlet boundary conditions*, AIMS Math., **8**, 1 (2023), 1172–1194.

[17] Z. LAADJAL, Q.-H. MA, N. ADJEROUD, *Lyapunov-type inequality for the Hadamard fractional boundary value problem on a general interval* $[a,b]$, J. Math. Inequal., **13**, 3 (2019), 789–799.

[18] P. LANGEVIN, *Sur la théorie du mouvement brownien (in French) [On the theory of Brownian motion]*, CR Acad. Sci. Paris, 146, (1908), 530–533.

[19] S. C. LIM, M. LI, AND L. P. TEO, *Langevin equation with two fractional orders*, Phys. Lett. A, **372**, 42 (2008), 6309–6320.

[20] S. C. LIM AND L. P. TEO, *The fractional oscillator process with two indices*, J. Phys. A: Math. Theor., **42**, 6, ID 065208 (2009), 1–34.

[21] L. LIZANA, T. AMBJORNSSON, A. TALONI, E. BARKAI, AND M. A. LOMHOLT, *Foundation of fractional Langevin equation: harmonization of a many-body problem*, Phys. Rev. E, **81**, ID 051118 (2010).

[22] A. LOZINSKI, R. G. OWENS, AND T. N. PHILLIPS, *The Langevin and Fokker-Planck equations in polymer rheology*, Handbook Numer. Analys., **16**, (2011), 211–303.

[23] A. M. LYAPUNOV, *Problème général de la stabilité du mouvement*, (French Translation of a Russian paper dated 1893), Ann. Fac. Sci. Univ. Toulouse, **2**, (1907), 27–247, Reprinted as Ann. Math. Studies, no. 17, Princeton, 1947.

[24] Q.-H. MA, C. MA, J. WANG, *A Lyapunov-type inequality for a fractional differential equation with Hadamard derivative*, J. Math. Inequal., **11**, 1 (2017), 135–141.

[25] S. K. NTOUYAS, B. AHMAD, *Lyapunov-type inequalities for fractional differential equations*, A survey, Surveys Math. Appl., **16**, (2021), 43–93.

[26] S. K. NTOUYAS, B. AHMAD, T. P. HORIKIS, *Recent developments of Lyapunov-type inequalities for fractional differential equations*, pp. 619–686. In: D. ANDRICA, T. RASSIAS (eds), *Differential and Integral Inequalities*, Springer Optimization and Its Applications, **151**, Springer, Cham, 2017.

[27] B. PACHPATTE, *On Lyapunov-type inequalities for certain higher order differential equations*, J. Math. Anal. Appl., **195**, 2 (1995), 527–536.

[28] S. PANIGRAHI, *Lyapunov-type integral inequalities for certain higher-order differential equations*, Electron. J. Diff. Equ., no. 28, (2009), 1–14.

[29] N. PARHI, S. PANIGRAHI, *On Lyapunov-type inequality for third order differential equations*, J. Math. Anal. Appl., **233**, 2 (1999), 445–460.

[30] J. P. PINASCO, *Lyapunov-type inequalities. With applications to eigenvalue problems*, Springer, New York, 2013.

[31] H.-G. SUN, Y. ZHANG, D. BALEANU, W. CHEN, Y.-Q. CHEN, *A new collection of real world applications of fractional calculus in science and engineering*, Commun. Nonlinear Sci. Numer. Simulat., **64**, (2018), 213–231.

[32] A. TIRYAKI, *Recent developments of Lyapunov-type inequalities*, Adv. Dyn. Syst. Appl., **5**, 2 (2010), 231–248.

[33] A. TIRYAKI, M. UNAL, D. CAKMAK, *Lyapunov-type inequalities for non-linear systems*, Math. Anal. Appl., **332**, 1 (2007), 497–511.

[34] C. TORRES, *Existence of solution for fractional Langevin equation: variational approach*, Electron. J. Qual. Theory Differ. Equ., no. 54, (2014), 1–14.

[35] M. URANAGASE AND T. MUNAKATA, *Generalized Langevin equation revisited: mechanical random force and self-consistent structure*, J. Physics A Math. Theor., **43**, 45, ID 455003 (2010), 1–11.

[36] Y. WANG, Q. WANG, *Lyapunov-type inequalities for fractional differential equations under multi-point boundary conditions*, J. Math. Inequal., **13**, 3 (2019), 611–619.

[37] A. WONGCHAROEN, B. AHMAD, S. K. NTOUYAS, J. TARIBOON, *Three-point boundary value problems for Langevin equation with Hilfer fractional derivative*, Adv. Math. Phys., ID 9606428 (2020), 1–11.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com