

STABILITY OF ADDITIVE FUNCTIONAL INEQUALITY IN VARIOUS NORMED SPACES

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Abstract. In this paper, we establish the general solution of the following functional inequality

$$\|2f(x) + 2f(y) + 2f(z) - f(x+y) - f(y+z)\| \leq \|f(x+z)\|,$$

and then investigate the generalized Hyers-Ulam stability of this inequality in Banach spaces and in non-Archimedean Banach spaces by using two different approaches.

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