AN APPLICATION OF KY FAN INEQUALITY: ON KULLBACK–LEIBLER DIVERGENCE BETWEEN A PROBABILITY DISTRIBUTION AND ITS NEGATION

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Abstract. The negation of a probability distribution has been defined in the context of belief theory by [13]. Since then, its generalizations and applications have been studied. Although a probability distribution and its negation have been studied separately in terms of entropy in the literature, the relative entropy (Kullback-Leibler divergence) between them has not been investigated. In this article, we investigate the difference between a probability distribution and its negation in terms of the Kullback-Leibler divergence. In particular, we establish some bounds for the Kullback-Leibler divergence in terms of logical entropy of a probability distribution. To do this, we not only use the generalized Ky Fan inequality but also provide an extension of the generalized Ky Fan inequality. As a result, this article provides a nice application of the generalized Ky Fan inequality in a special topic in information science.

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