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SOME NEW IMPROVEMENTS OF YOUNG'S INEQUALITIES

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Abstract. In this paper, we obtain some improvements and generalizations of Young's inequalities as the following:

(1) If $b \geqslant a$, we can get

$$\frac{(a\nabla_v b)^m - (a\sharp_v b)^m}{(a\nabla_\tau b)^m - (a\sharp_\tau b)^m} \leqslant \frac{v(1-v)}{\tau(1-\tau)};$$

(2) If $b \leqslant a$, we can get

$$\frac{(a\nabla_v b)^m - (a\sharp_v b)^m}{(a\nabla_\tau b)^m - (a\sharp_\tau b)^m} \geqslant \frac{v(1-v)}{\tau(1-\tau)}$$

for $m \in N_+$ and $0 < v \leqslant \tau < 1$. In addition, we obtain new result of Young's inequality by using the expansions of the functions $(1-v)+vx-x^v$ with $0 < x < 2$.

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