EXISTENCE AND ASYMPTOTIC BEHAVIOR OF SQUARE–MEAN S–ASYMPTOTICALLY PERIODIC SOLUTIONS FOR STOCHASTIC EVOLUTION EQUATION INVOLVING DELAY

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Abstract. This paper studies the stochastic evolution equations with finite delay. By means of the compact semigroup theory and Schauder fixed point theorem, the existence of square-mean S-asymptotically periodic mild solutions is obtained under certain growth conditions. In addition, using the contraction mapping principle and Gronwall integral inequality, the uniqueness and global asymptotic stability of the square-mean S-asymptotically periodic mild solutions are discussed. Finally, an example is given to illustrate our abstract results.

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