

A LYAPUNOV–TYPE INEQUALITY FOR A CLASS OF HIGHER–ORDER FRACTIONAL BOUNDARY VALUE PROBLEMS

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Abstract. This work presents a new Lyapunov-type inequality for a class of higher-order fractional boundary value problem of the fractional Caputo Fabrizio differential equation subject to fractional integral boundary conditions. The derived result is applied to the fractional Sturm-Liouville problem in establishing a lower bound for the eigenvalues. We also provide the necessary condition for nonexistence of the non-trivial solution of the fractional boundary value problem.

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