A NEW PARTIAL ORDER BASED ON CORE
PARTIAL ORDER AND STAR PARTIAL ORDER

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Abstract. In this paper, we introduce a new partial order called CS partial order, which is based on the core partial order and star partial order. We give some characteristics of CS partial order by using polar decomposition. Then we apply the polar-like decomposition to introduce another new partial order called WC (weak CS) partial order, which is an extension of the CS partial order. We illustrate the relationships of the two partial orders with some well-known partial orders, such as minus, Löwner, GL, CL partial order. At the end of the paper, the application of CS and WC partial orders is briefly discussed from the perspective of quantum physics.

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