A THEOREM AND AN ALGORITHM INVOLVING MUIRHEAD’S INEQUALITY

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Abstract. Let \( a, b \in \mathbb{R}^n \) be column vectors, and \((u, v)\) be the inner product of vectors \( u \) and \( v \) on \( \mathbb{R}^n \). Let \( G \subset \text{GL}(n, \mathbb{R}) \) be a compact matrix group. For \( A \in G \) and a continue function \( f \) on \( G \), the integral \( \int_G f(A)\,dA \) is the invariant integral of the compact group \( G \). In this paper, we study the inequality

\[
\forall x \in \mathbb{R}^n \quad \int_G e^{(Aa, x)}\,dA \geq \int_G e^{(Ab, x)}\,dA.
\]

We prove that the above inequality holds if and only if \( b \in \text{Conv}(Ga) \). This work follows a series of results, that is, Muirhead (1903), Hardy, Littlewood and Pólya (1932), Rado (1952), Daykin (1971), Kimelfeld (1995) and Schulman (2009). Furthermore, we construct an determining algorithm when \( G \) is finite. Compared with other effective algorithms, this one is symbolic and easy to implement on computer.


Keywords and phrases: Compact matrix groups, inequality, convex hull.

REFERENCES