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MEAN SQUARE OF QUADRATIC HECKE CHARACTER SUM

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Abstract. Assuming the Generalized Riemann Hypothesis (GRH) and the Riemann Hypothesis (RH), we consider the sum $S(X,Y)$ in the Gaussian field $\mathbb{Q}(i)$, where the sum is taken over all primary primes. The primary tools employed include the Poisson sum and Mellin inversion techniques. The method used in this paper differs significantly from the classical approach which relies on the partial sum method, thereby facilitating a simpler calculation within our proof. Furthermore, we present a valid asymptotic formula that is applicable when both X and Y are of comparable size.

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