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SHARP INEQUALITIES OF IYENGAR–MADHAVA RAO–NANJUNDIAH TYPE INCLUDING $\cos\left(\frac{x}{\sqrt{3}}+ax^r\right)$

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Abstract. In this paper, for $0<x<\frac{\pi}{2}$ and $r>0$, we consider the following Iyengar-Madhava Rao-Nanjundiah type inequality;

$$\cos\left(\frac{x}{\sqrt{3}}+\alpha x^r\right)<\frac{\sin x}{x}<\cos\left(\frac{x}{\sqrt{3}}+\beta x^r\right).$$

Our main theorems shows that α and β depend on $r>0$, and if $0<r<3$ then

$$\beta=\left(\frac{2}{\pi}\right)^r\left(-\frac{\pi}{2\sqrt{3}}+\arccos\frac{2}{\pi}\right)$$

and if $r>4$ then

$$\alpha=\left(\frac{2}{\pi}\right)^r\left(-\frac{\pi}{2\sqrt{3}}+\arccos\frac{2}{\pi}\right).$$

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