

## IMPROVEMENTS OF THE WEIGHTED HERMITE–HADAMARD INEQUALITY AND APPLICATIONS TO MEAN INEQUALITY

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**Abstract.** This paper aims to characterize the function appearing in the weighted Hermite-Hadamard inequality. We provide improved inequalities for the weighted means as applications of the obtained results. Modifications of the weighted Hermite-Hadamard inequality are also presented. Our results contain some exciting inequalities and extensions of the known results.

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