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DIMENSION–FREE ESTIMATES FOR HARDY–LITTLEWOOD MAXIMAL FUNCTIONS WITH MIXED HOMOGENEITIES

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Abstract. We mainly study the dimension-free L^p -inequality of the Hardy-Littlewood maximal functions with mixed homogeneities

$$M_*^G f(x,y) = \sup_{t>0} \frac{1}{|G|} \left| \int_G f(x-tu, y-t^2 v)dudv \right|,$$

where G is a bounded, closed and symmetric convex subset of $\mathbb{R}^{d+1}$. When G is in the isotropic position, we prove that there is a constant C_p independent of d such that

$$\left\| M_*^G f \right\|_{L^p(\mathbb{R}^{d+1})} \leqslant C_p(L(G)) \|f\|_{L^p(\mathbb{R}^{d+1})},$$

for $\frac{3}{2} < p \leqslant \infty$, where $L(G)$ is a constant associated with G.

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Paper JMI-17-51

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