

NONEXISTENCE AND EXISTENCE OF POSITIVE GROUND STATE SOLUTIONS FOR GENERALIZED QUASILINEAR SCHRÖDINGER EQUATIONS

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Abstract. This paper is concerned with a class of generalized quasilinear Schrödinger equations which have appeared from plasma physics, as well as high-power ultrashort laser in matter. Combining the variable replacement and the Schauder-Tychonoff fixed point theorem, we establish the nonexistence and existence of positive radial ground state solutions for this problem.

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