

ASYMPTOTICS FOR RANDOM–TIME RUIN PROBABILITY OF A RISK MODEL WITH DIFFUSION, CONSTANT INTEREST FORCE AND NON–STATIONARY ARRIVALS

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Abstract. Consider an insurance risk model with diffusion, constant interest force and non-stationary arrivals, where the claims arrive according to a non-stationary process satisfying a large deviation principle. The asymptotic formula for the random-time ruin probability is obtained if the claim-size distribution is subexponential. Furthermore, with a certain dependence structure among claim sizes, the formula still holds if the claim-size distribution belongs to the class with long tails and dominatedly varying tails.

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