QUASI–CONVEX AND Q–CLASS FUNCTIONS

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Abstract. Convex functions and their variants have played a significant role in the literature. In this article, we investigate two important related classes, namely quasi-convex and Q-class functions. We will show that these two classes satisfy similar but different properties as those fulfilled by convex functions. Our discussion will include refinements of known inequalities, super-additivity behavior, Jensen-Mercer inequality, and other related results. Among many other results, we show that an increasing quasi-convex function $f : [0,\infty) \to \mathbb{R}$ satisfies the inequality

$$\frac{f(a)+f(b)}{2} \leqslant f\left(\frac{a+b}{2}\right) + \frac{1}{2}f(a+b), \quad (a,b > 0),$$

while a Q-class function with $f(0) \leqslant 0$ satisfies the super-additive inequality

$$f(a) + f(b) \leqslant \frac{(a+b)^2}{ab}f(a+b), \quad (a,b > 0)$$

similar to convex functions.

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