

## A NEW ISOPERIMETRIC-TYPE INEQUALITY AND ITS STABILITY

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*Abstract.* In this paper, we discuss the isoperimetric-type inequalities for the closed convex curve in the Euclidean plane  $\mathbb{R}^2$ . The main result of this paper is that we establish a family of parametric inequalities. By adjusting the parameters, we can derive some well-known isoperimetric inequalities and some new improved isoperimetric-type inequalities. Furthermore, we investigate the stability property of the main inequality.

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