

THE THIRD-ORDER HERMITIAN TOEPLITZ DETERMINANT FOR SOME CLASSES OF ANALYTIC FUNCTIONS

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Abstract. Sharp lower and upper bounds are found of the second and third-order Hermitian Toeplitz determinants for some classes of analytic functions.

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