ON GENERALIZED SINGULAR NUMBER OF
POSITIVE MATRIX OF τ MEASURABLE OPERATORS

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Abstract. Let $(\mathcal{M}, \tau)$ be a semi-finite von Neumann algebra, $L_0(\mathcal{M})$ be the set of all τ-measurable operators. We studied generalized singular numbers of 2×2 positive matrices with entries in $L_0(\mathcal{M})$. We proved the equivalence of several inequalities associated with these generalized singular numbers and gave symmetric norm's version of this results, i.e., we extend the related inequalities of 2×2 positive semi-definite block matrices in [1, 5] to the 2×2 positive matrices of τ-measurable operators case.

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